Stackelberg Game for Utility-Based Cooperative Cognitive Radio Networks

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ABSTRACT

With the development of cognitive radio technologies, dynamic spectrum access becomes a promising approach to increase the efficiency of spectrum utilization and solve spectrum scarcity problem. Under dynamic spectrum access, unlicensed wireless users (secondary users) can dynamically access the licensed bands from legacy spectrum holders (primary users) on an opportunistic basis. While most primary users in existing works assume secondary transmissions as negative interference and don't actively involve them into the primary transmission, in this paper, motivated by the idea of cooperative communication, we propose a cooperative cognitive radio framework, where primary users, aware of the existence of secondary users, may select some of them to be the cooperative relay, and in return lease portion of the channel access time to them for their own data transmission. Secondary users cooperating with primary transmissions have the right to decide their payment made for primary user in order to achieve a proportional access time to the wireless media. Both primary and secondary users target at maximizing their utilities in terms of their transmission rate and revenue/payment. This model is formulated as a Stackelberg game and a unique Nash Equilibrium point is achieved in analytical format. Based on the analysis we discuss the condition under which cooperation will increase the performance of the whole system. Both analytical result and numerical result show that the cooperative cognitive radio framework is a promising framework under which the utility of both primary and secondary system are maximized.

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General Terms

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1. INTRODUCTION

The demand for wireless spectrum has been growing rapidly with the dramatic development of the mobile telecommunication industry in recent years, thus spectrum scarcity is becoming a severe problem that the whole industry must face to. Both academic and regulatory bodies are recognizing that traditional fixed spectrum allocation can be very inefficient, considering that most of the time, bandwidth that has been allocated is not used and the corresponding channel is idle, which forms spectrum holes. In order to fully utilize the scarce spectrum resources, with the development of cognitive radio technologies, dynamic spectrum access becomes a promising approach to increase the efficiency of spectrum usage, which allows unlicensed wireless users (secondary users) to dynamically access the licensed bands from legacy spectrum holders (primary users) on a negotiated or an opportunistic basis.

Most of the existing works on dynamic spectrum access fall into one of the following two directions according to the side information known by primary users. The first which is also the one that most current research focused on, assumes that primary user is unaware of the presence of secondary users, and secondary users are allowed to access the channel only if their transmission will not cause interference to the primary users, which can be achieved by spectrum sensing or power control. In another word, in this approach, secondary users are totally transparent to primary users. The second direction assumes that licensed users are aware of the existence of secondary users which are using their allocated frequency band. However, they have higher priority on accessing the channel and have the right to improve their own revenue by charging secondary users for using their own licensed frequency band.

One important observation is that, even in the latter type, the primary user do not leverage secondary user for their own transmissions. There is no inter-operation between the primary system and the secondary system on data transmission and decoding. In wireless networks, due to the channel fading, direct transmission from the primary transmitter to the primary receiver is sometimes severely damaged and thus suffers bad performance in terms of data rate and outage probability. Motivated by the recently emerged physical layer technology called cooperative communication, we know that if some users, either other primary users or secondary users, which have better channel conditions, are leveraged as the cooperative relays for the primary transmission, the transmission rate can be dramatically increased and the outage probability is decreased by exploiting cooperative diversity. In particular, if other primary users are not in the suitable location where they are easy to help, or they have a heavy traffic load which make them no idle time for extra data forwarding, then suitable secondary users are better to be selected as the cooperative relays to improve the performance of primary transmission. Meanwhile, as the primary user's rate is increased, to serve certain amount of primary traffic, time occupied by the primary transmission is decreased. As a result, secondary users gain more opportunity to access the wireless channel and transmit more data of secondary system. Therefore, by exploiting cooperation between primary system and secondary system, both systems can increase their own interest and a win-win situation can be achieved. We call this kind of cooperating network as *cooperative cognitive radio network* (CCRN) which is a new cognitive radio paradigm.

In regard to the aforementioned cooperative cognitive radio networks, there is only one existing work [13], in which primary users lease their spectrum to secondary users for a fraction of time and in exchange, they get the cooperative transmission power from secondary users. The primary user targets at maximizing transmission rate while secondary users compete with each other when accessing the channel. However, for most primary services, when the required traffic demand is satisfied, primary systems have no interest to increase their transmission rate any more, instead, they want to achieve certain benefit in other format, for example revenue, which is more interesting to them. Based on this observation, in this paper, we propose a novel cooperative cognitive radio framework, where primary user actively selects some appropriate secondary users and involves them into the primary transmission as cooperative relays. In return, chances to access the wireless media are given to the selected secondary relays. To achieve the access chance, secondary relays should conduct cooperative transmission for primary user and meanwhile make a payment, which is proportional to the access time, to the primary user. For primary user, the target is to maximize its utility which depends on both its transmission rate and revenue obtained from the secondary users. For secondary users, the target is to decide how much they should pay for the primary user so as to achieve its maximal transmission rate.

Assumed that both primary and secondary users are rational and selfish, which are interested in maximizing their own utility, the considered framework can be analyzed by game theory. In addition, the model is characterized by a hierarchical structure, where one agent (the primary link) optimizes its strategy (leased time and amount of cooperation) based on the knowledge of the effects of its decision on the behavior of a second agent (the secondary network). A convenient analytical model to study this scenario is provided by Stackelberg games. By backward induction, we can prove that there exists a unique Nash Equilibrium (NE) point for this Stackelberg game under certain constraints. We then give an analytical result of the NE point and analyze the constraint under which primary user tends to share a portion of transmission time and secondary users tends to cooperate. Promisingly, the results give guidance to a cooperative protocol which is easily to be implemented in the cooperative cognitive radio networks to achieve the cooperation among primary and secondary users. Both analytical and numerical results show that primary user and secondary user have the motivation to cooperate with each other under certain circumstances and the performance of both systems are dramatically improved if they cooperate.

The main contributions of the paper are as follows. 1) a novel cooperative cognitive radio framework is proposed to enable the primary user to involve secondary users as the cooperative relay and in return, the secondary users achieve the opportunity to access the wireless channel for their own data transmission. A payment mechanism, where secondary users pay charges to primary user in order to achieve the access opportunity, is designed in the framework to further motivate the cooperation. 2) We formulate the hierarchical framework as a Stackelberg game, with primary user acting as the leader and secondary users as the follower. We prove that there exists a unique Nash Equilibrium for the game and give the analytical result of it and its corresponding constraints. 3) Cooperative protocol is designed and implemented based on the analytical result. 4) Numerical analysis shows that under our framework, both primary and secondary systems achieve better performance in terms of transmission rate and revenue when doing cooperation.

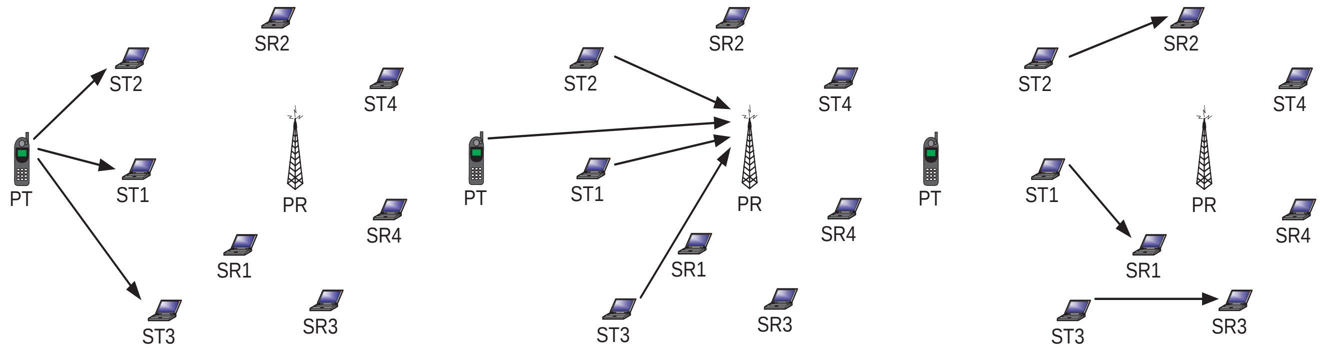
The rest of the paper is organized as follows. In Section 2, we give a detailed description of the system model. The Stackelberg game model and equilibrium analysis are presented in Section 3. We discuss the implementation protocol of the model in Section 4 and performance analysis is given in Section 5. Related work is reviewed in Section 6. Finally in Section 7 are the conclusions.

2. SYSTEM MODEL

In this section, we describe the model of cooperative cognitive radio network and the main system parameters.

We consider the system sketched in Figure 1, where a primary (licensed) transmitter PT communicates with the intended receiver PR. In the same spectrum band, a secondary (unlicensed) network $\mathcal{S}_{total}$, composed of K transmitters-receivers pairs $\{\mathrm{ST}_i, \mathrm{SR}_i\}_{i=1}^{K}$, is seeking to exploit possible transmission opportunities. There is a predefined traffic requirement in terms of transmission rate R_{req} for primary transmission pair. No traffic requirement is imposed for secondary network. Each secondary link accesses the channel and transmits data as much as possible in best-effort manner. Primary transmitter may select several secondary transmitters from the secondary network to behave as the cooperative relays, and in return, give them the chance to access the channel which is assumed to be occupied only by primary system. Secondary transmitters selected can gain the wireless channel only if they cooperate with the primary link and meanwhile make a certain amount of payment to the primary system.

The primary transmitter PT grants the use of the bandwidth to a subset $\mathcal{S}_{total}$ of $|\mathcal{S}| = k \leq |\mathcal{S}_{total}| = K$ secondary nodes in exchange for cooperation so as to improve the quality of the communication link to its receiver PR. In particular, the primary decides whether to use the entire slot for direct transmission to PR or to employ cooperation. In the latter case, a fraction α of the slot $0 \leq \alpha \leq 1$ is used for primary transmission from PT to PR. Further more, this portion of slot is divided into two subslots according to a parameter β ($0 \leq \beta \leq 1$). α, β are parameters dynami-



(a) Primary transmission (b) Cooperative transmission (c) Secondary transmission

Figure 1: System model for cooperative cognitive radio networks: (a) in 1st ($\alpha\beta$) fraction of slot, primary user transmit data; (b) in 2nd ($\alpha(1-\beta)$) fraction of slot, both primary and secondary users transmit data cooperatively; (c) in remaining $(1-\alpha)$ fraction of slot, secondary users access channel to transmit their own data. $(0 \leq \alpha, \beta \leq 1)$

cally selected by primary transmitter. The first subslot is of duration $\alpha\beta$ unit time and is dedicated to the transmission of PT to all cooperative transmitters ST_i in subset $\mathcal{S}$ (Figure 1(a)). The second subslot is of duration $\alpha(1-\beta)$ unit time and in this subslot, both PT and all cooperative relays in subset S cooperatively transmit data to primary receiver PR (Figure 1(b)). The remaining $1-\alpha$ unit time of slot is for secondary transmitter-receiver pairs to access the wireless channel and transmit data for secondary system. In this fraction of slot, k secondary transmitters in set $\mathcal{S}$ access the channel in time-division multiplexing access (TDMA) mode(Figure 1(c)). The time duration each secondary transmitter ST_i achieves for its own data transmission is proportional to the payment it made for the primary user c_i, $t_i = (1-\alpha)c_i/\Sigma_{j=1}^{k} c_j$. It is assumed that the cost is always positive and up-bounded by a maximum payment $\bar{c}$, i.e. $0 \leq c \leq \bar{c}$.

The channels between nodes are modelled as independent proper complex Gaussian random variables, invariant within each slot, but generally varying over the slots (i.e., Rayleigh block-fading channels). We use the following notation to denote the instantaneous fading channels in each block: h_P denotes the complex channel gain between primary transmitter PT and primary receiver PR; $h_{PS,i}$ denotes the channel gain between PT and secondary transmitter ST_i; $h_{SP,i}$ denotes the channel gain between ST_i and PR; $h_{S,i}$ denotes the channel gain between the ith secondary transmitter and receiver pair ST_i and SR_i, for any $i = 1, ..., K$.

For simplicity, it is assumed that there is no power control, both primary transmitter and secondary transmitters are transmitting at fixed power level. Denote P_P to be the power of PT and $P_{S,i}$ to be that of ST_i.

Therefore, the transmission rate of each link can be calculated as follows. For primary transmission, in the first subslot, PT broadcasts packets to all ST_i in set $\mathcal{S}$, in order for all the receivers to successfully decode the data, the transmission rate is dominated by the worst channel $h_{PS,i}$ in the subset $i \in \mathcal{S}$ as

$$R_{PS}(\mathcal{S}) = \log_2(1 + \frac{\min_{i\in\mathcal{S}} |h_{PS,i}|^2 P_P}{N_0}). \quad (1)$$

In the second subslot, both primary transmitter and secondary transmitters cooperatively transmit primary data to primary receiver. Assuming the receiver exploit maximum ratio combining before decoding the signal, the effective signal-to-noise ratio (SNR) equals to the sum of all the SNRs of each transmitter. Therefore, the achievable rate of the cooperative link is given by

$$R_{SP}(\mathcal{S}) = \log(1 + \frac{|h_P|^2 P_P}{N_0} + \sum_{i\in\mathcal{S}} \frac{|h_{SP,i}|^2 P_S}{N_0}). \quad (2)$$

The overall achievable primary rate of the decode-and-forward cooperative transmission equals to the minimum transmission rate of the two stages,

$$R_P(\alpha, \beta, \mathcal{S}) = \min\{\alpha\beta R_{PS}(\mathcal{S}), \alpha(1-\beta)R_{SP}(\mathcal{S})\}. \quad (3)$$

The transmission rate of secondary link can be calculated directly by the SNR of the signal received by the corresponding secondary receiver,

$$R_{S,i} = \log(1 + \frac{|h_{S,i}|^2 P_S}{N_0}), \qquad \forall i \in \mathcal{S}. \quad (4)$$

3. GAME THEORY ANALYSIS

In this section, we define the utility function for both primary users and secondary users, formulate the cooperative cognitive radio networks as a Stackelberg game and analyze the equilibrium existence and uniqueness of the game.

Based on the model described in Section 2, we assume that both primary users and secondary users are rational and selfish. As primary user is licensed user and therefore has a higher priority on using the frequency band, it has the right to decide the parameters α, β and $\mathcal{S}$, so as to maximize its own utility in terms of both traffic rate and revenue. Secondary transmitter in $\mathcal{S}$, which is competitive with other secondary links, decides only the payment it is willing to make, under the pre-decided α, β with the target of maximizing transmission rate without making too much payment. Therefore, it is a typical two-stage leader-follower game which can be analyzed under Stackelberg game framework, where, primary link, the leader of the game, optimizes its strategy based on the knowledge of the effects of its decision on the behavior of the followers (secondary links).

3.1 Utility Functions

Now we give a concrete expression of utility functions, basing on which, the existence and uniqueness of the Nash Equilibrium is analyzed.

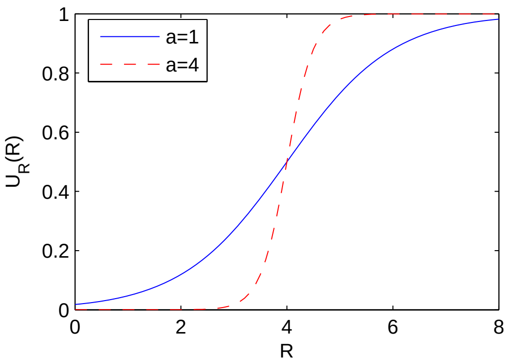


Figure 2: Primary user's satisfactory function $U_R(R(\alpha, \beta, \mathcal{S}))$ of the transmission rate

The utility function of primary user is defined to be the weighted sum of the utility function of primary user's transmission rate and the revenue it collects from the secondary relays.

$$U_P = w_P U_R(R_P(\alpha, \beta, \mathcal{S})) + \sum_{i \in \mathcal{S}} c_i, \tag{5}$$

where w_P is the equivalent revenue per unit data rate utility contributes to the overall utility, which is a predefined parameter. $U_R(R_P(\alpha, \beta, \mathcal{S}))$, data rate utility, is a measure of primary user's degree of satisfaction and can be modeled as a sigmoid function of the achievable transmission rate (Figure 2).

$$U_R(R_P(\alpha, \beta, \mathcal{S})) = \frac{1}{1 + e^{-a(R_P(\alpha,\beta,\mathcal{S}) - R_0)}}, \tag{6}$$

where R_0 is the primary user's traffic requirement, and a decides the steepness of the satisfactory curve. Sigmoid function [15] has been widely used to approximate the user's satisfaction with respect to service qualities or resource allocation [9, 14]. There is a threshold R_0 below which the user has very limited satisfaction (convex segment) and above which his satisfaction rapidly reaches an asymptotic value (concave segment).

Secondary users target at maximizing transmission rate of their own data under a reasonable payment. Therefore, the utility function of each secondary link is defined to be its achievable transmission rate in equivalent revenue format minus the payment it makes for the primary user.

$$u_i = \frac{w_s(1-\alpha)c_i R_i}{\sum_j c_j} - c_i, \tag{7}$$

where w_s is the equivalent revenue per unit transmission rate contributes to the overall utility. As secondary users work in best effort manner, and no requirement is imposed on their transmission, the utility functions are simply linear with the transmission rates they are able to achieve, which are proportional to the payment they are willing to pay.

3.2 Secondary User Payment Selection Game

Given the utility functions defined in previous section, we use backward induction to analysis the performance of the Stackelberg game. Given α, β and $\mathcal{S}$ decided by primary user, several secondary users in the cooperative relay set $\mathcal{S}$ compete with each other to maximize its own utility by selecting its payment, which forms a noncooperative payment selection game (NPG) $G = [\mathcal{S}, \{C_i\}, \{u_i(\cdot)\}]$, where $\mathcal{S}$ is the player set selected by primary user, C_i is the strategy set, and $u_i(\cdot)$ is the utility function of user i. In particular, each player ST_i with $i \in \mathcal{S}$, selects its strategy within the strategy space $\mathcal{C} = \mathbf{C} = [C_i]_{i\in\mathcal{S}} : 0 \le c_i \le \bar{c}$ to maximize its utility function $u_i(c_i, \mathbf{c}_{-i})$. We first analyze the NE existence and uniqueness of NPG.

DEFINITION 1. *A payment vector $\mathbf{c} = (c_1, \cdots, c_k)$ is a Nash equilibrium of the NPG $G = [\mathcal{S}, \{C_i\}, \{u_i(\cdot)\}]$ if, for every $i \in \mathcal{S}$, $u_i(c_i, \mathbf{c}_{-i}) \ge u_i(c_i', \mathbf{c}_{-i})$ for all $c_i' \in P_i$, where $u_i(c_i, \mathbf{c}_{-i})$ is the resulting payment for the ith user given the other players' payment selection result $\mathbf{c}_{-i}$.*

THEOREM 1. *A Nash equilibrium exists in the NPG, $G = [\mathcal{S}, \{C_i\}, \{u_i(\cdot)\}]$.*

PROOF. The following result is obtained from [2].

PROPOSITION 1. *A Nash equilibrium exists in game $G = [\mathcal{S}, \{C_i\}, \{u_i(\cdot)\}]$, if for all $i \in \mathcal{S}$:*
1)C_i is a nonempty, convex, and compact subset of some Euclidean space $\Re^N$.
2)$u_i(\mathbf{c})$ is continuous in $\mathbf{c}$ and concave in c_i.

Strategy space is defined to be $\mathcal{C} = \mathbf{C} = [C_i]_{i\in\mathcal{S}} : 0 \le c_i \le \bar{c}$. So it is a nonempty, convex and compact subset of the Euclidean space $\Re^k$.

$$u_i = \frac{w_s(1-\alpha)c_i R_i}{\sum_{j\in\mathcal{S}} c_j} - c_i, \tag{8}$$

from which, u_i is obviously continuous in $\mathbf{c}$. Now we take the second order derivative with respect to c_i to prove its concavity.

$$\frac{\partial u_i}{\partial c_i} = \frac{w_s(1-\alpha)R_i \sum_{j\in\mathcal{S}, j\ne i} c_j}{(\sum_{j\in\mathcal{S}} c_j)^2} - 1 \tag{9}$$

$$\frac{\partial^2 u_i}{\partial^2 c_i} = \frac{-2w_s(1-\alpha)R_i \sum_{j\in\mathcal{S}, j\ne i} c_j}{(\sum_{j\in\mathcal{S}} c_j)^3} < 0 \tag{10}$$

The second order derivative of u_i with respect to c_i is always less than 0, therefore, $u_i(\mathbf{c})$ is concave in c_i.

According to Proposition 1, a Nash equilibrium exists in game NPG. □

THEOREM 2. *The game NPG has a unique equilibrium.*

PROOF. By Theorem 1, we know that there exists an Nash equilibrium in NPG. Let $\mathbf{c}$ denote the Nash equilibrium in the NPG. By definition, the Nash equilibrium has to satisfy $\mathbf{c} = \mathbf{r}(\mathbf{c})$ where $\mathbf{r}(\mathbf{c}) = (r_1(\mathbf{c}), r_2(\mathbf{c}), \cdots, r_k(\mathbf{c}))$. $r_i(\mathbf{c})$ is the best response function of player i given the payment selection of other players $r_i(\mathbf{c}) = r_i(\mathbf{c}_{-i})$. The key aspect of the uniqueness proof is to realize that the best-response correspondence $\mathbf{r}(\mathbf{c})$ is a *standard* function [16]. A function is said to be standard if it satisfies the following properties;

- positivity: $\mathbf{r}(\mathbf{c}) > 0$;

$$r_i(\mathbf{c}) = c_i^* = \begin{cases} 0 & \text{if } \sum_{j\neq i} c_j \geq (1-\alpha)w_s R_i \\ \sqrt{(1-\alpha)w_s R_i \sum_{j\neq i} c_j} - \sum_{j\neq i} c_j & \text{if } \sum_{j\neq i} c_j < (1-\alpha)w_s R_i, \sqrt{(1-\alpha)w_s R_i \sum_{j\neq i} c_j} - \sum_{j\neq i} c_j < \overline{c} \\ \overline{c} & \text{if } \sqrt{(1-\alpha)w_s R_i \sum_{j\neq i} c_j} - \sum_{j\neq i} c_j \geq \overline{c} \end{cases} \quad (11)$$

- monotonicity: if $\mathbf{c} \geq \mathbf{c}'$ then $\mathbf{r}(\mathbf{c}) \geq \mathbf{r}(\mathbf{c}')$;
- scalability: for all $\mu > 1$, $\mu\mathbf{r}(\mathbf{c}) > \mathbf{r}(\mu\mathbf{c})$.

It is shown in [16] that the fixed point $\mathbf{c} = \mathbf{r}(\mathbf{c})$ is unique for a standard function. Therefore, the Nash equilibrium of NPG is unique.

Secondary user i's utility function u_i is concave with respect to c_i, therefore, the best-response correspondence is achieved when the first derivative of u_i with c_i equals to 0, i.e.,

$$\frac{\partial u_i}{\partial c_i} = \frac{w(1-\alpha)R_i \sum_{j\neq i} c_j}{(\sum_{j\in\mathcal{S}} c_j)^2} - 1 = 0 \quad (12)$$

Solve equation (12), we have different solutions under various constraints, as shown in (11),

Assuming that the constraints for the second case are satisfied,

$$\sum_{j\neq i} c_j < (1-\alpha)w_s R_i \quad (13)$$

and

$$\sqrt{(1-\alpha)w_s R_i \sum_{j\neq i} c_j} - \sum_{j\neq i} c_j < \overline{c}, \quad (14)$$

the best-response correspondence is calculated as

$$r_i(\mathbf{c}) = \sqrt{(1-\alpha)w_s R_i \sum_{j\neq i} c_j} - \sum_{j\neq i} c_j \quad (15)$$

We first prove the positivity of $r_i(\mathbf{c})$. Given the constraint $\sum_{j\neq i} c_j < (1-\alpha)w_s R_i$, the best-response function is always positive,

$$\begin{aligned} r_i(\mathbf{c}) &= \sqrt{(1-\alpha)w_s R_i \sum_{j\neq i} c_j} - \sum_{j\neq i} c_j \\ &> \sqrt{\sum_{j\neq i} c_j \cdot \sum_{j\neq i} c_j} - \sum_{j\neq i} c_j \\ &= 0 \end{aligned}$$

As for monotonicity, $r_i(\mathbf{c})$ is a quadratic function of the term $\sqrt{\sum_{j\neq i} c_j}$. Therefore, when $\sum_{j\neq i} c_j \leq \frac{1}{4}(1-\alpha)w_s R_i$, $\mathbf{r}(\mathbf{c})$ is monotonically increasing function, i.e., $\mathbf{r}(\mathbf{c})$, when $\mathbf{c} \geq \mathbf{c}'$.

As for scalability, we have,

$$\begin{aligned} & \mu r_i(\mathbf{c}) - r_i(\mu\mathbf{c}) \\ = \; & \mu(\sqrt{(1-\alpha)w_s R_i \sum_{j\neq i} c_j} - \sum_{j\neq i} c_j) \\ & -(\sqrt{(1-\alpha)w_s R_i \mu \sum_{j\neq i} c_j} - \mu \sum_{j\neq i} c_j) \\ = \; & (\mu - \sqrt{\mu})\sqrt{(1-\alpha)w_s R_i \sum_{j\neq i} c_j} \end{aligned}$$

For $\forall \mu > 1$, we have $\mu - \sqrt{\mu} > 0$, therefore, (11) is positive and $\mu\mathbf{r}(\mathbf{c}) > \mathbf{r}(\mu\mathbf{c})$ is always satisfied.

In conclusion, the best-response correspondence $\mathbf{r}(\mathbf{c})$, which is positive, monotonic and scalable, is a standard function. Therefore, there exits a unique NE point for NPG $G = [\mathcal{S}, \{C_i\}, \{u_i(\cdot)\}]$. □

THEOREM 3. *The unique equilibrium for game NPG is given by*

$$c_i^* = w_s(1-\alpha)(k-1)[\sum_{j\in\mathcal{S}} \frac{1}{R_j} - \frac{k-1}{R_i}]/(\sum_{j\in\mathcal{S}} \frac{1}{R_j})^2. \quad (16)$$

PROOF. Solve the equations set (12) which consists of k equations when $i = 1, 2, \cdots, k$, the result is given by (16). □

Substituting (16) into (13) and (14), we can rewrite the constraints as follows,

$$\sum_{j\in\mathcal{S}} \frac{1}{R_j} - \frac{2(k-1)}{R_i} > 0 \quad (17)$$

$$\overline{c} > w_s(1-\alpha)(k-1)[\sum_{j\in\mathcal{S}} \frac{1}{R_j} - \frac{k-1}{\max_{i\in\mathcal{S}} R_i}]/(\sum_{j\in\mathcal{S}} \frac{1}{R_j})^2. \quad (18)$$

These constraints will be used by the primary user to select optimal cooperative relay set $\mathcal{S}^*$.

3.3 Maximizing Primary User's Utility

Based on the analytical result of the secondary user's payment selection game, the leader of the Stackelberg game, primary user, can optimize its strategy $(\alpha, \beta, \mathcal{S})$ in order to maximize its revenue according to (5), being aware that its decision will affect the strategy selected by the Stackelberg follower (secondary users). Substituting (16) into (5), the utility of primary user is given by

$$U_P = \frac{w_p}{1 + e^{-a(R_P(\alpha,\beta,S)-R_0)}} + \frac{w_s(1-\alpha)(k-1)}{\sum_i (1/R_i)}, \quad (19)$$

where $R_P(\alpha, \beta, S)$ is the effective cooperative transmission rate of the primary user, which is given by (3).

PROPOSITION 2. *U_P achieve maximization if and only if $\beta = R_{SP}(\mathcal{S})/(R_{SP}(\mathcal{S}) + R_{PS}(\mathcal{S}))$.*

PROOF. According to (3), cooperative transmission rate for primary user $R_P(\alpha, \beta, \mathcal{S})$ equals to the minimum rate of the two stages. Moreover, $R_P(\alpha, \beta, \mathcal{S})$ is the minimum of an increasing function of β, $\alpha\beta R_{PS}(\mathcal{S})$, and an decreasing function of β, $\alpha(1-\beta)R_{SP}(\mathcal{S})$, and therefore maximization of $R_P(\alpha, \beta, \mathcal{S})$ is achieved when the two terms are equal:

$$\alpha\beta R_{PS}(\mathcal{S}) = \alpha(1-\beta)R_{SP}(\mathcal{S}) \quad (20)$$

According to (19), U_P is an increasing function of $R_P(\alpha, \beta, \mathcal{S})$, therefore, maximization of U_P is achieved when $R_P(\alpha, \beta, \mathcal{S})$

is maximized, which requires the satisfaction of (20). According to (20), we have,

$$\beta^* = R_{SP}(\mathcal{S})/(R_{SP}(\mathcal{S}) + R_{PS}(\mathcal{S})). \quad (21)$$

Proof is finished. □

We now turn to the optimal parameter selection of the overall Stackelberg game. The optimal parameters α^* and β^* selected by the primary user is given and the conditions under which, cooperation is beneficial for the overall system (i.e. $0 < \alpha^* < 1$) is given in the following theorem.

THEOREM 4. *When the following conditions*

$$B \leq A/4 \quad (22)$$

$$e^{-A+aR_0} \leq \frac{A - 2B + \sqrt{A^2 - 4AB}}{2B} \leq e^{aR_0} \quad (23)$$

are satisfied, primary user maximizes its utility function if and only if parameters α and β are set to be the following optimal values,

$$\alpha = \alpha^* = [aR_0 - \ln(\frac{A - 2B + \sqrt{A^2 - 4AB}}{2B})]/A, \quad (24)$$

$$\beta = \beta^* = R_{SP}(\mathcal{S})/(R_{SP}(\mathcal{S}) + R_{PS}(\mathcal{S})), \quad (25)$$

in which,

$$A = aw_p R_{PS}(\mathcal{S}) R_{SP}(\mathcal{S})/(R_{SP}(\mathcal{S}) + R_{PS}(\mathcal{S})), \quad (26)$$

$$B = w_s(k-1)/\sum_i (1/R_i). \quad (27)$$

PROOF. Substituting (21) into (19), and calculate the first order derivative of U_P in respect with α, we have,

$$\frac{\partial U_P}{\partial \alpha} = \frac{AX}{(X+1)^2} - B \quad (28)$$

where, A is given by (26), B is given by (27), and X is given by

$$X = e^{-\alpha A + aR_0}. \quad (29)$$

When $B \geq A/4$, the first order derivative of U_P in respect with α is always negative,

$$\frac{\partial U_P}{\partial \alpha} = \frac{AX}{(X+1)^2} - B \leq A/4 - B \leq 0. \quad (30)$$

Noticed that $0 \leq \alpha \leq 1$, U_P is maximized at $\alpha = 0$. This result implies that primary user prefer providing all of the timeslot to secondary users for their access, without transmitting any data of its own. The reason which leads the primary user to such an extreme decision is that w_p is small and therefore, utility increased by additional rate is not comparable with that by additional payment charged from the secondary users.

When $B < A/4$, with the increase of α, U_P is a decreasing function of α at first, then, an increasing function, and at last a decreasing function. There is a local minimum point α_1 and a local maximum point α_2 when U_P is defined in the whole real number field. Both α_1 and α_2 can be calculated by assigning the first order derivative to be 0.

$$\alpha_1 = [aR_0 - \ln(\frac{A - 2B - \sqrt{A^2 - 4AB}}{2B})]/A, \quad (31)$$

$$\alpha_2 = [aR_0 - \ln(\frac{A - 2B + \sqrt{A^2 - 4AB}}{2B})]/A. \quad (32)$$

The maximization of U_P is achieved either in the boundary of domain area, or in the local maximization point. What we are interested in are two things. One is the criteria under which the cooperation is beneficial, in another word, when the optimal α^* satisfies $0 < \alpha^* < 1$. The other is the optimal value of α^* and the optimal value of the utility function.

We now answer the first question. The optimal parameter α^* is achieved in the interior area of the domain instead of the boundary, if and only if the local maximum point falls into the interior range of α's domain, which is given by

$$0 < \alpha_2 < 1, \quad (33)$$

and, local maximum point is also the global maximum,which is given by

$$U_P(0) \leq U_P(\alpha_2). \quad (34)$$

It is obvious that (33) is equivalent to (23), and (34) is equivalent to (24). The conditions when the cooperation is beneficial is proven.

Under the conditions derived in (22)and(23), it is easy to see that when α equals to the local maximum point, which is also the global maximum point, the utility function of the primary user is maximized.

$$\alpha^* = \alpha_2 = [aR_0 - \ln(\frac{A - 2B + \sqrt{A^2 - 4AB}}{2B})]/A, \quad (35)$$

$$U_P^* = U_P(\alpha^*). \quad (36)$$

□

4. IMPLEMENTATION PROTOCOL

In this section, we will propose a cooperation protocol to dynamically select the parameters in the cooperative cognitive radio system based on the analytical results of the Stackelberg game.

According to (24) and (25), besides a, w_p, w_s, which are predefined parameters for the system, optimal slot division α^* and β^* are also functions of the transmission rates between PT and ST_i, $R_{PS,i}$, between ST_i and PR, $R_{SP,i}$, between PT and PR, R_P, and that between ST_i and SR_i, R_i, which are changing all the time with real time channel condition. Meanwhile, α^* and β^* are also functions of the selected relay set $\mathcal{S}$, which complies with the criteria (17) and (18).

In order to calculate the real time transmission rate, primary transmitter periodically collects channel conditions from the primary receiver and each secondary transmitter. $|h_P|^2$ and $|h_{SP,i}|^2$ are collected from PR, while $|h_{PS,i}|^2$ $|h_{S,i}|^2$ are collected from each ST_i. Based on the calculated transmission rates, primary transmitter then, from the universal set $\mathcal{S}_{total}$, enumerates all the possible cooperative relay set $\mathcal{S}$, which satisfy the relay set selection criteria (17) and (18). Based on each possible relay set $\mathcal{S}$, the optimal time division parameters $\alpha^*(\mathcal{S})$, $\beta^*(\mathcal{S})$ and overall utility of the primary user $U_P^*(\mathcal{S})$ can be calculated. From all possible sets, the one that maximizes primary user's utility function $\mathcal{S}^* = \text{argmax}_{\mathcal{S}} U_P^*(\mathcal{S})$ is selected to be the optimal relay set, and optimal time division α^* and β^* are set to be the corresponding parameters under the optimal set respectively, $\alpha^* = \alpha^*(\mathcal{S}^*)$, $\beta^* = \beta^*(\mathcal{S}^*)$.

When the optimal parameters is calculated, primary transmitter piggybacks the value of the optimal parameters α^*, β^* and $\mathcal{S}^*$, which are useful for secondary users to calculate their optimal payments c_i, behind the data packets and broadcasts them to all secondary users. After receiving these parameters, secondary users calculate c_i distributively according to (16). Noticed that c_i depends only on α, k, R_i and $\sum_{j \in \mathcal{S}} 1/R_j$, only the values of α, k and $\sum_{j \in \mathcal{S}} 1/R_j$ need to be piggybacked by the primary user. After decoding α, k and $\sum_{j \in \mathcal{S}} 1/R_j$, c_i can be calculated and the access time is known for each secondary transmitter. In particular, c_i depends on the sum of all the inverse of each secondary link's rate, instead of each individual rate, therefore, one secondary user don't need to know any other's channel condition, which saves a lot of message exchange and allows the protocol to be implemented more distributively.

5. SIMULATION RESULTS

In this section, we show some numerical results to unveil the impact of different system characteristics on the optimal cooperation scheme selection. We consider a simple geometrical model where secondary nodes are all placed at approximately the same normalized distance d $(0 < d < 1)$ from the primary transmitter PT and $1 - d$ from the primary receiver PR. Considering a path loss model, assume the average power gain of the primary channel equals to 1, $E|h_P|^2 = 1$, then, the average channel gain between primary and secondary users can be given as follows, $E|h_{PS,i}|^2 = 1/d^{\eta}$, $E|h_{SP,i}|^2 = 1/(1-d)^{\eta}$. For secondary network, we assume the average channel gain of each link $E|h_{S,i}|^2 = 0.8$. Both primary user and secondary users transmit at a fixed power level without power control. The signal-to-noise ratio (SNR) is $SNR = P_P/N_0 = 10$. The other parameters used in the simulation are set as follows, unless explicitly stated otherwise: the rate-weighted parameters for primary and secondary users are $w_p = 0.3$ and $w_s = 0.15$ respectively; steepness parameter a for rate-utility function $U_R(R)$ is $a = 1$ and the required data rate for primary user $R0 = 3.6$; each secondary user has an upper-bounded payment $\bar(c) = 0.1$; the number of secondary transmitters is $K = 6$, from which primary user randomly selects k $(2 \leq k \leq K)$ users to form the relay set.

Figure 3 shows the optimal parameters α^* and β^*, averaged over fading distribution, versus the normalized distance d under various numbers of secondary relays. With the increase of normalized distance d, the broadcast transmission rate from RT to ST_i is decreased, while the cooperative transmission rate from ST_i to PR is increased. To receive certain amount of data and forward the same amount out, more time is needed for the first broadcast stage and less is needed for the second cooperation stage. Therefore, β^* increases when the normalized distance d becomes larger, which also agrees with the analysis result given in (refe18). α^* is also increased when the unified distance becomes larger, but with a smaller increasing rate, which also complies with the analysis result shown in (24).

Figure 4 and Figure 5 show the utility function under different schemes versus the normalized distance d and number of selected secondary relays k. U_P denotes the utility function of the optimal scheme, in which primary user leases some timeslot for secondary user and leverages them to transmit cooperatively. U_0 denotes the primary user's utility function when $\alpha = 0$, which implies that all the primary user's channel time is given to the secondary users to receive payment without sending any of its own data (leasing spectrum). U_d denotes utility of the scheme when no cooperation is leveraged and all the channel time is used for the licensed service. The simulation shows that when k is fixed to $k = 5$, utility function of the optimal cooperation scheme outperforms that of leasing spectrum scheme by 30%, and increases that of direct transmission dramatically by 150%, as shown in Figure 4. The benefit is brought by the appropriate trade-off between improved transmission rate of the primary user and higher revenue received from the secondary users. Figure 5 shows the value of primary user's utility function of different schemes under various numbers of relays k, given the uniform distance $d = 0.3$. When k increases from 2 to 6, both U_P and U_0 are increased due to the improved cooperative transmission rate $R_{SP,i}$, while U_d keeps constant and independent of number of relays k. U_P and U_0 increase simultaneously, so that U_P keeps achieving at least 30% gain compared with U_0.

Figure 6 shows the relationship between the optimal variables α^*, β^* and primary user's required transmission rate R_0, where uniform distance d is fixed to $d = 0.3$ and number

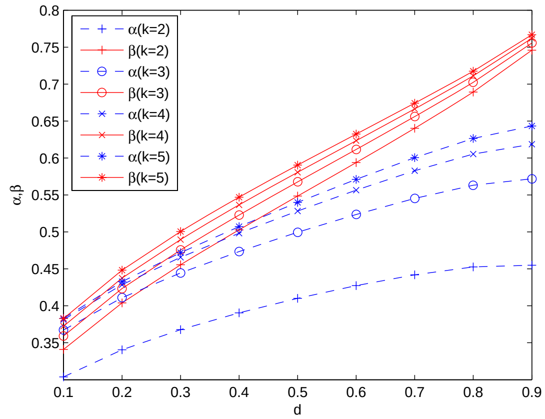


Figure 3: Optimal α^* and β^* versus the normalized distance d and number of relays k

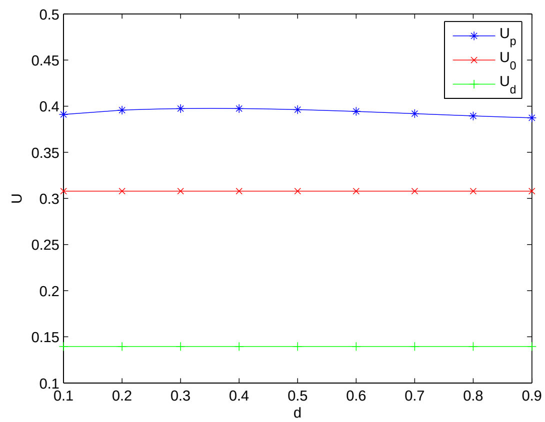


Figure 4: Primary user's utility function of different schemes versus the normalized distance d

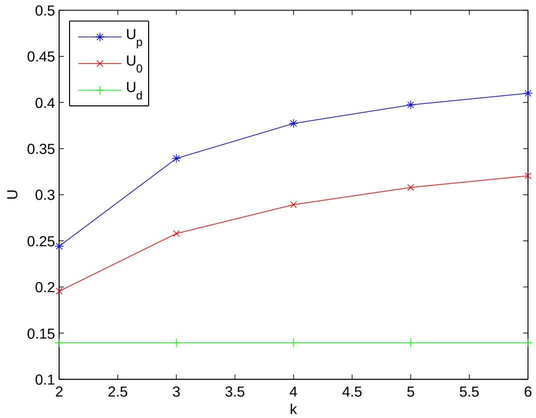


Figure 5: Primary user's utility function of different schemes versus number of relays k

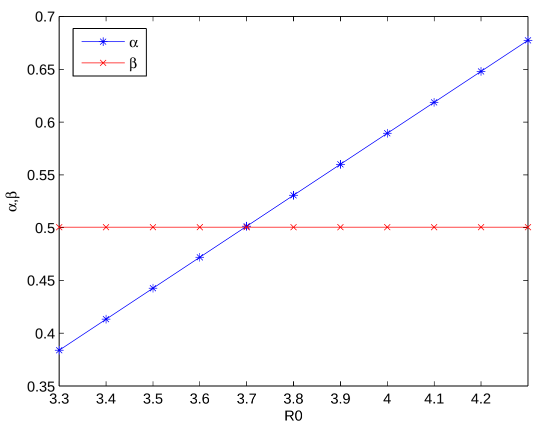


Figure 6: Optimal α^* and β^* versus required primary rate R_0

of relays k to be $k = 5$. The result shows that α^* increases approximately linearly with the increase of required transmission rate R_0, as larger R_0 requires more time spent on transmission of primary user's data, while β^* keeps independent of required rate R_0 and remains constant. This simulation result also complies with the analytical result given by (24) and (25).

6. RELATED WORKS

Cognitive radio systems can be divided into three paradigms: underlay, overlay and interweave paradigms [1, 8]. In underlay paradigm, the concurrent legacy and cognitive transmissions may occur only if the interference generated by the cognitive devices at the legacy receivers is below certain acceptable threshold. In interweave paradigm, cognitive radio can periodically monitor the radio spectrum and opportunistically communicates over spectrum holes where the primary users are not active. This paper lies in the overlay paradigm, where cognitive user is allowed to transmit simultaneously with legacy user, if the interference to legacy user can be offset by using part of the cognitive user's power to relay the legacy user's message. Actually, in overlay paradigm, there is a natural necessary to exploit cooperative diversity in the cognitive radio systems, because the spatial and user diversity provided by the cognitive relays can dramatically enhance the performance of primary systems. However, most existing works [3, 7, 10] focus on the information theoretic analysis, which only give the capacity region for the overlay system, without considering implementation problems. In contrary, our work seeks to present a practical framework to implement the cooperative cognitive radio system and design a payment mechanism to motivate the cooperation, whose performance is analyzed using game theory.

Game theory has been advocated as an appropriate framework to study the competitive spectrum access in cognitive networks [11]. [6] provides a game theoretical overview of dynamic spectrum sharing from three aspects: analysis of network users' behaviors, efficient dynamic distributed design and optimality analysis. The dynamic spectrum sharing problem can be formulated as various game models, e.g., single-stage non-cooperative dynamic spectrum sharing game [12], single-stage cooperative dynamic spectrum sharing game [4], multi-stage dynamic game and auction game [5]. In our paper, as the model is a hierarchical model, where primary user acts as leader and secondary users act as followers, we formulate the problem as a Stackelberg game and analyze it using backward induction to calculate the Nash equilibrium of the game.

So far as we know, there is only one work [13] focusing on using game theory to analyze the performance of cooperation in cognitive radio networks. The authors propose and investigate a solution for spectrum leasing based on the idea that secondary nodes can earn spectrum access in exchange for cooperation with the primary link. The primary user in [13] targets at maximizing its transmission rate, while in our work, primary user can improve its benefit by either increasing transmission rate when the traffic demand is not satisfied, or collecting a higher revenue from secondary users when the achievable rate is high. Our model is more rational because each primary system has certain traffic demand and when its demand is satisfied, it is willing to enlarge its benefit in other format and have no more interest in increasing the rate. Another difference between [13] and our work lies on the access scheme of secondary users. Their transmission scheme amounts to an interference channel with distributed power control, where all the secondary links are allowed to access the channel at the same time within an SNR constraint. However, under the assumption that all the cooperative transmitters are located almost at the same place,which is assumed in [13], the SNR constraint is difficult to guarantee. Therefore, we adopt TDMA as the secondary users access model and design a payment mechanism to divide the time between multiple secondary users.

7. CONCLUSIONS

In this paper, we propose a novel cooperative cognitive radio framework which enable the primary user to involve secondary users as the cooperative relay and in return, the secondary users achieve the opportunity to access the wireless channel for their own data transmission. A payment mechanism, where secondary users pay charges to primary user in order to achieve the access opportunity, is designed in the framework to further motivate the cooperation. By formulating the novel model as a Stackelberg game, we prove that there exists a unique Nash Equilibrium for the game and derive the analytical result of it. Fortunately, the analytical results lead to an implementation protocol where no channel information of other secondary users is needed. Numerical analysis shows that under our framework, both primary and secondary systems achieve better performance in terms of transmission rate and revenue when doing cooperation.

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9. REFERENCES

[1] I. M. A. Goldsmith, S. A. Jafar and S. Srinivasa. Breaking spectrum gridlock with cognitive radios: An information theoretic perspective. *Proceedings of the IEEE, invited*, to appear 2008.

[2] G. Debreu. A social equilibrium existence theorem. *Proc. Nat. Acad. Science*, 38.

[3] N. Devroye, P. Mitran, and V. Tarokh. Achievable rates in cognitive radio channels. *Information Theory, IEEE Transactions on*, 52(5):1813–1827, May 2006.

[4] Z. Han, Z. Ji, and K. Liu. Fair multiuser channel allocation for ofdma networks using nash bargaining solutions and coalitions. *Communications, IEEE Transactions on*, 53(8):1366–1376, Aug. 2005.

[5] J. Huang, R. Berry, and M. Honig. Auction-based spectrum sharing. *Mobile Networks and Applications*, 11(3):405–408, June 2006.

[6] Z. Ji and K. Liu. Cognitive radios for dynamic spectrum access - dynamic spectrum sharing: A game theoretical overview. *Communications Magazine, IEEE*, 45(5):88–94, May 2007.

[7] J. Jiang and Y. Xin. On the achievable rate regions for interference channels with degraded message sets. *Information Theory, IEEE Transactions on*, 54(10):4707–4712, Oct. 2008.

[8] P. J. Kolodzy. Cognitive radio fundamentals. *SDR Forum, Singapore*, April.

[9] H. Lin, M. Chatterjee, S. K. Das, and K. Basu. Arc: An integrated admission and rate control framework for competitive wireless cdma data networks using noncooperative games. *IEEE Transactions on Mobile Computing*, 4(3):243–258, 2005.

[10] I. Maric, R. Yates, and G. Kramer. Capacity of interference channels with partial transmitter cooperation. *Information Theory, IEEE Transactions on*, 53(10):3536–3548, Oct. 2007.

[11] G. R. Neel J, Reed JH. The role of game theory in the analysis of software radio networks. *SDR forum technical conference, San Diego, California*, Nov.

[12] G. Owen. *Game Theory*. Academic Press, 2005.

[13] O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz. Spectrum leasing to cooperating secondary ad hoc networks. *Selected Areas in Communications, IEEE Journal on*, 26(1):203–213, Jan. 2008.

[14] G. Stamoulis, D. Kalopsikakis, and A. Kyrikoglou. Efficient agent-based negotiation for telecommunications services. *Global Telecommunications Conference, 1999. GLOBECOM '99*, 3:1989–1996 vol.3, 1999.

[15] D. H. von Seggern. *CRC Standard Curves and Surfaces with Mathematica, Second Edition (Chapman & Hall/Crc Applied Mathematics and Nonlinear Science)*. Chapman & Hall/CRC, 2006.

[16] R. Yates. A framework for uplink power control in cellular radio systems. *Selected Areas in Communications, IEEE Journal on*, 13(7):1341–1347, Sep 1995.